National
Qualifications
2024

2024 Mathematics Paper 1 Non-calculator

Advanced Higher

Question Paper Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate’s response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•1 begin differentiation [1]	•1 $-\operatorname{cosec}^2 3x$	**2**
			•2 apply chain rule [2]	•2 $-3\operatorname{cosec}^2 3x$	

Notes:

1. Where a candidate equates y to the derivative, •1 is still available.
2. At •2, accept $-\operatorname{cosec}^2 3x \times 3$.

Commonly Observed Responses:

Where a candidate writes $\cot 3x$ as $\frac{\cos 3x}{\sin 3x}$:

$\frac{-3\sin 3x \sin 3x - \ldots}{\sin^2 3x}$ or $\frac{\ldots - 3\cos 3x \cos 3x}{\sin^2 3x}$ award •1

$\frac{-3}{\sin^2 3x}$ award •2

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 evidence of use of product rule with one term correct [1,2]	•3 $5(4x-7)^{\frac{1}{2}} + \ldots$ or $\ldots + 5x \times \frac{1}{2} \times 4(4x-7)^{-\frac{1}{2}}$	**2**
			•4 complete differentiation	•4 $5(4x-7)^{\frac{1}{2}} + 10x(4x-7)^{-\frac{1}{2}}$	

Notes:

1. Except where it results from rearrangement of a correct answer, if a candidate produces one term only, award 0/2.
2. Where a candidate equates the derivative to the original function, •3 is not available (see COR).

Commonly Observed Responses:

Candidate equates derivative to original function:

$f(x) = 5x(4x-7)^{\frac{1}{2}}$

$= 5(4x-7)^{\frac{1}{2}} + 10x(4x-7)^{-\frac{1}{2}}$

Do not award •3.

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•[1] calculate modulus or argument [1,2]	•[1] $\sqrt{2}$ or $\frac{\pi}{4}$	**2**
			•[2] write in polar form [1,2]	•[2] $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$	

Notes:

1. Accept arguments expressed in degrees provided a degree symbol appears at least once in part (a) or (b); otherwise withhold •[2].

2. Any working leading to calculation of the argument (and the modulus) must be consistent (General Marking Principle (n)).

Commonly Observed Responses:

	(b)		•[3] apply de Moivre's theorem to argument [1,2,3]	•[3] $\cos\frac{8\pi}{4}+i\sin\frac{8\pi}{4}$	**2**
			•[4] complete process [4,5]	•[4] 16	

Notes:

1. For the award of •[3], there must be a single argument. Disregard the form of the modulus.
2. It is not sufficient at •[3] for the argument to be expressed as a variable.
3. Where a candidate has produced a zero argument at •[2], •[3] is available only where 0 is explicitly multiplied by 8.
4. Where a candidate has produced a modulus for z equal to ± 1, •[4] is not available.
5. At •[4], do not accept $\ldots\left(\cos 2\pi + i\sin 2\pi\right)$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] interpret geometric sequence [1,2] •[2] calculate common ratio [2,3]	•[1] $ar^2 = 36$ and $ar^4 = 16$ •[2] $\frac{2}{3}$	**2**

Notes:

1. Award •[1] for $r^2 = \frac{16}{36}$
2. For the award of •[1], there must be some evidence of strategy, eg 36 24 16. For a statement of the answer without justification, award •[2] only.
3. There is no requirement for an explicit rejection of a negative answer at •[2].

Commonly Observed Responses:

$r = \frac{16}{36} = \sqrt{\frac{16}{36}} = \frac{4}{6} = \frac{2}{3}$ award •[2] only

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] calculate first term	•[3] 81	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[4] show condition is satisfied [1,2,3]	•[4] $\left\lvert\frac{2}{3}\right\rvert < 1$ or equivalent	**1**

Notes:

1. For •[4] $\frac{2}{3}$ may be replaced with a letter consistent with their answer to (a). However, in the case where a candidate obtains a value in (a) outside the open interval $(-1,1)$, •[4] is available only where they also acknowledge that there is no sum to infinity.
2. For •[4], accept an equivalent statement in words. However, if a candidate uses the term "between", it must be explicitly stated that it is strictly between.
3. Where the answer contains incorrect (rather than insufficient) information (before, between or after correct information), •[4] is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(d)		$\bullet^5$ calculate sum to infinity [1]	$\bullet^5$ 243	**1**

Notes:

1. Where an incorrect value is calculated in (a), $\bullet^5$ is available only where that value satisfies the condition for convergence.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ find determinant or adjunct [1,2] •² find A^{-1} [1,2]	•¹ $\det A = 7$ or $\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}$ •² $\frac{1}{7}\begin{pmatrix} 3 & -1 \\ -11 & 6 \end{pmatrix}$	**2**

Notes:

1. For correct answer with no working, award 2/2.
2. Where the determinant has not been explicitly identified, •¹ may be awarded for $\frac{1}{7}\begin{pmatrix} \cdots & \cdots \\ \cdots & \cdots \end{pmatrix}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ determine M in terms of A^{-1} and B [1] •⁴ find matrix M [2,3]	•³ $M = A^{-1}B$ •⁴ $\begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$	**2**

Notes:

1. Where a candidate has written $\frac{B}{A}$, award •³ only if they subsequently write or calculate $A^{-1}B$.

2. At •⁴, accept $\frac{1}{7}\begin{pmatrix} -7 & 7 \\ 14 & -21 \end{pmatrix}$

3. At •⁴, the only acceptable multiplications are $A^{-1}B$ or BA^{-1}.

Commonly Observed Responses:

COR A

For $BA^{-1} = \frac{1}{7}\begin{pmatrix} -45 & 22 \\ -37 & 17 \end{pmatrix}$, award •⁴.

COR B

For candidates who use simultaneous equations:

Award •³ for four equations, or a pair of equations with solutions, eg

$$\begin{aligned} 6a + c &= -4 \\ 11a + 3c &= -5 \\ 6b + d &= 3 \\ 11b + 3d &= 2 \end{aligned}$$

or eg

$$\begin{aligned} 6a + c &= -4 \\ 11a + 3c &= -5 \end{aligned}$$ leading to $a = -1$, $c = 2$

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] expression for $f(-x)$ [1,2] •[2] justify that the function is odd [1,2]	•[1] $f(-x)=(-x)^3-(-x)$, stated or implied •[2] $-x^3+x8$ leading to $=-f(x)$ or $-(x^3-x)$ odd	2

Notes:

1. Where a candidate has used an exclusively graphical approach, a sketch including roots and stationary points is required for the award of •[1]. For •[2], reference must made to half-turn symmetry about the origin.
2. Award 0/2 for a numerical approach.

Commonly Observed Responses:

	(b)		•[3] equate second derivative to 0 [1,2,3] •[4] consider sign of $f''(x)$ for $x<0$ and $x>0$ and state conclusion [4]	•[3] $6x=0$ •[4] $x>0 \Rightarrow f''(x)>0$ and $x<0 \Rightarrow f''(x)<0$ $\therefore$ POI	2

Notes:

1. Given that the second derivative exists for all x, it is sufficient to consider only a zero second derivative.
2. Do not withhold •[3] where a candidate states that points of inflection occur when the second derivative equals zero.
3. Where a candidate does not explicitly equate $6x$ to zero, •[3] may be awarded for $f''(x)=0$ provided they also write $f''(x)=6x$.
4. May be awarded where $f''(x)$ and $f''(-x)$ have been calculated for a specific value of x close to 0 and shown to have opposite signs.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] obtain matrix	•[1] $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[2] describe effect [1]	•[2] reflection in the line $y = x$.	**1**

Notes:

1. Reference to reflection (in/across/along) $y = x$ must appear.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[3] correct order for multiplication [1] •[4] complete multiplication [2,3]	•[3] $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ •[4] $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	**2**

Notes:

1. Do not withhold •[3] for incorrect information prior to $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

2. •[4] is not available if a candidate incorrectly identified $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ at •[1].

3. **Beware** of incorrect working leading to $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ at •[4].

Commonly Observed Responses:

Incorrect order of multiplication:

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] apply product rule [1]	•[1] $2xy + x^2\frac{dy}{dx}$ or $4y^2 + 8xy\frac{dy}{dx}$	3
			•[2] complete differentiation [1]	•[2] $2xy + x^2\frac{dy}{dx} + 4y^2 + 8xy\frac{dy}{dx} = 0$	
			•[3] find expression for $\frac{dy}{dx}$ [2,3]	•[3] $\frac{dy}{dx} = \frac{-2xy - 4y^2}{x^2 + 8xy}$	

Notes:

1. Terms need not be simplified at •[1] or •[2].
2. Award •[3] only where $\frac{dy}{dx}$ appears more than once after the candidate has completed their differentiation.
3. Withhold •[3] if there is further incorrect simplification of $\frac{dy}{dx}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] equate expression for $\frac{dy}{dx}$ to 0 [1,2,3]	•[4] $\frac{-2xy - 4y^2}{x^2 + 8xy} = 0$	3
			•[5] state linear relationship between y and x at stationary point [2,4,5,6]	•[5] eg $y = \frac{-x}{2}$, $x = -2y$, $x + 2y = 0$	
			•[6] determine coordinates of stationary point [2,5,6,7]	•[6] $(-4, 2)$	

Notes:

1. Award •[4] for substitution of $\frac{dy}{dx} = 0$ into the equation at •[2].
2. Where a candidate has failed to differentiate the RHS of the original equation, only •[4] is available.
3. At •[4], accept $-2xy - 4y^2 = 0$.
4. For the award of •[5], the relationship need not be simplified.
5. Where a candidate equates the denominator to zero, •[5] and •[6] are not available regardless of the processing of the numerator.
6. Disregard the appearance of $y = 0$.
7. For the award of •[6], there must be a linear relationship between y and x at •[5].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•¹ differentiate	•¹ $du = 2\sec^2 2x dx$ or $\frac{du}{dx} = 2\sec^2 2x$	4
			•² determine new limits and begin to rewrite integrand [1,2]	•² $\int_0^1 \ldots du$	
			•³ complete integrand [2,3,4,5]	•³ $\int_0^1 \frac{1}{2}\sqrt{u}\, du$	
			•⁴ evaluate [5]	•⁴ $\frac{1}{3}$	

Notes:

1. Any working leading to calculation of new limits at •² must be consistent (General Marking Principle (n)).
2. Except as indicated in COR A, candidates must produce an integral including the correct new limits and du at some point for the award of •².
3. Except as indicated in COR A, disregard the omission of du and/or limits for the award of •³.
4. Where the integrand contains terms in x, •³ is still available provided these terms are clearly and correctly "cancelled out".
5. Where candidates attempt to integrate an expression containing both u and x, where x is either inside the integrand or taken outside as a constant, •³ and •⁴ are not available.

Question		Generic scheme	Illustrative scheme	Max mark
8.	(continued)			

Commonly Observed Responses:

COR A

No limits in new integral and return to original variable:

$\int \frac{1}{2}\sqrt{u}\,du$	award •³ provided du appears at this stage
$\left[\frac{1}{3}(\tan 2x)^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$	award •²

COR B

Wrong limits in new integral and return to original variable:

$\int_0^{\frac{\pi}{8}} \dots du$	do not award •²
$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$	award •³
$\left[\frac{1}{3}(\tan 2x)^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$ leading to $\frac{1}{3}$	award •⁴

COR C

Wrong limits in new integral and new limits used in evaluation:

$\int_0^{\frac{\pi}{8}} \dots du$	do not award •²
$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$	award •³
$\left[\frac{1}{3}u^{\frac{3}{2}}\right]_0^{1}$ leading to $\frac{1}{3}$	award •⁴

COR D

Wrong limits in new integral and no return to original variable:

$\int_0^{\frac{\pi}{8}} \dots du$	do not award •²
$\int_0^{\frac{\pi}{8}} \frac{1}{2}\sqrt{u}\,du$	award •³
$\left[\frac{1}{3}u^{\frac{3}{2}}\right]_0^{\frac{\pi}{8}}$	do not award •⁴

[END OF MARKING INSTRUCTIONS]